

Statistical Inference

Take-Home Midterm

↑ Student name ↑

Instructor: Bjørn Kjos-Hanssen

Disclaimer: It is essential to write legibly and *show your work*. If your work is absent or illegible, and at the same time your answer is not perfectly correct, then no partial credit can be awarded. Completely correct answers which are given without justification may receive little or no credit.

During this exam, you are permitted to use calculators, notes, and the textbook, but not to collaborate with others.

Score: _____

Problem	Score/4
1	
2	
3	
4	
5	
Total/20	

Score: _____

Problem 1. Suppose Z is a standard normal random variable and you are given the fact (from a table) that

$$\mathbb{P}(Z \leq 1.96) = 97.5\%.$$

Suppose Y is a normal random variable with mean μ and variance σ^2 . You draw $n = 3$ samples from the distribution of Y and observe $(Y_1, Y_2, Y_3) = (3, 1, 4)$. Find a 95% confidence interval for μ .

You can leave the answer as an arithmetical expression. For example, “ $(3 - 5/\sqrt{2}, 3 + 5/\sqrt{2})$ ” is an answer in an acceptable form.

Score: _____

Problem 2. *Bounce rate* is an Internet marketing term used in web traffic analysis. It represents the percentage of visitors who enter the site and “bounce” (leave the site) rather than continue viewing other pages within the same site. Suppose that for the web site *math.hawaii.edu*, the overall bounce rate is 55.23% on average.

On some recent weekend days the average bounce rate was observed to be 58.94%. The total number of visitors on the observed days was 1,792. Do these results provide strong evidence that the bounce rate is higher on weekends?

- a) State appropriate null and alternative hypotheses.
- b) Find a test statistic. Show the formula that you use, not just the numerical answer.
- c) Find the p-value for your test.
- d) Write a sentence or two explaining the meaning of the p-value you calculated in part c, in the context of this problem.
- e) Are you able to reject the null hypothesis (use significance level .05)?
- f) Write a sentence explaining your conclusion in the context of the problem.

Score: _____

Problem 3. Consider the following two estimators for the variance σ^2 of a random variable Y with known mean μ . Suppose (Y_1, \dots, Y_n) is a random sample from the distribution of Y .

$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^n (Y_i - \mu)^2,$$

$$\hat{\theta}_2 = \frac{1}{n-1} \sum_{i=1}^n (Y_i - \mu)^2.$$

Is one of $\hat{\theta}_1, \hat{\theta}_2$ necessarily unbiased (and if so, which one)? Justify.

Score: _____

Problem 4. Show that the statistic $\hat{\sigma}^2 = \sum_{i=1}^n Y_i^2$ is sufficient for σ^2 if Y_1, \dots, Y_n is a random sample from a normal pdf with $\mu = 0$.

Score: _____

Problem 5. Suppose X is a normally distributed random variable with mean 0 and standard deviation σ .

- (a) Write down the probability density function of X .
- (b) Write down the likelihood function $L(\sigma)$ for the sample

$$(X_1, \dots, X_3) = (x_1, \dots, x_3) = (0, 1, -1).$$

- (c) Find the maximum likelihood estimate of the standard deviation σ for the sample in (b).